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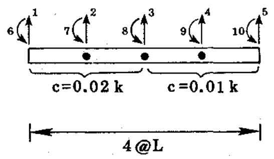

Fig. 3 Free-free beam with nonclassical damping.

a whole has nonproportional damping. The system stiffness matrix K is singular and $n_r = 2$. For each rigid-body mode, $C\phi_r^D = \mathbf{0}$. Therefore, this system requires both regular and generalized state rigid-body modes, so the state rigid-body modes are given by Eq. (27), where

$$\Phi_r^D = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & L & 2L & 3L & 4L & 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T$$

Summary

When the equations of motion of a structure having rigid-body freedom are cast in state-variable form, generalized state rigid-body modes may be required. The equations governing these generalized eigenvectors have been given, and examples of undamped and damped structures have been presented.

Acknowledgments

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References

[1]Craig, R. R., *Structural Dynamics—An Introduction to Computer Methods,* Wiley, New York, 1981.

[2]Craig, R. R., and Ni, Z., "Component Mode Synthesis for Non-Classically-Damped Systems," *Journal of Guidance, Control, and Dynamics,* Vol. 12, No. 4, 1989, pp. 577–584.

[3]Chen, C.-T., *Linear System Theory and Design,* Holt, Rinehart and Winston, New York, 1984, pp. 37, 38.

Stability Condition for Flexible Structure Control with Mode Residualization

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Introduction

FOR the structural control or flutter suppression problem, the dynamic model of a structurally flexible system necessarily includes a large number of modes (theoretically infinite). However, only a limited number of modes can be accommodated in controller design. One of the methods for obtaining a reduced-order controller is to use the formalism of mode residualization. This is extensively applied in the flutter suppression[1] and structural control[2,3] problems. This formulation of the reduced-order model is obtained by assuming that the residual modal states of the system quickly reach steady state values. The reduced-order controller design is then based on this simplified model. Although such a controller usually works well on the reduced-order system, it may not stabilize the actual closed-loop system. This is, of course, the most important evaluation criterion for a control law selection.

In this Note, we develop the general idea of mode residualization for a high-order, flexible structure and obtain results for the operation of an observer-based controller, based on the reduced-order model, in closed-loop with the actual system. By applying the perturbation theory, we derive a new stability condition on the basis of the frequency domain representation, which guarantees the stability of the overall closed-loop system even when the sensors and actuators are not collocated. In the flutter suppression or other structural control problems, the natural frequency, mode shape, and damping characteristics of modes are all known imprecisely. Thus, parameter uncertainty may cause the system to be unstable. Addressing this issue, we also present a simple robust stability condition, which insures the overall stability so that the controlled system will not be destabilized by parametric perturbations.

System Description

Consider the partitioned state-space representation of linear time-invariant systems having the following form

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{12}x_2(t) + B_1u(t) \tag{1a}$$

$$\dot{x}_2(t) = A_{21}x_1(t) + A_{22}x_2(t) + B_2u(t) \tag{1b}$$

$$y(t) = C_1x_1(t) + C_2x_2(t) \tag{1c}$$

in which $x_1(t) \in R^{n_1}$ and $x_2(t) \in R^{n_2}$ represent the model state vector and residual mode state vector, respectively; $u(t) \in R^m$ is the control vector; $y(t) \in R^r$ is the output vector; A_{11}, A_{12}, A_{21} and A_{22} are, respectively, $n_1 \times n_1$, $n_1 \times n_2$, $n_2 \times n_1$ and $n_2 \times n_2$ plant matrices; B_1 and B_2 are, respectively, $n_1 \times m$ and $n_2 \times m$ input matrices; C_1 and C_2 are, respectively, $r \times n_1$ and $r \times n_2$ output matrices.

If A_{22} is nonsingular, by assuming small changes in the residual modal states $x_2(t)$ (i.e., $\dot{x}_2(t) \to 0$), we have the following reduced-order system

$$\dot{\bar{x}}_1(t) = A\bar{x}_1(t) + Bu(t) \tag{2a}$$

$$\bar{y}(t) = C\bar{x}_1(t) + Du(t) \tag{2b}$$

where

$$A = A_{11} - A_{12}A_{22}^{-1}A_{21}, \qquad B = B_1 - A_{12}A_{22}^{-1}B_2$$

$$C = C_1 - C_2A_{22}^{-1}A_{21}, \qquad D = -C_2A_{22}^{-1}B_2$$

Note that although the full-order system [Eq. (1)] does not have a direct-feed term [D matrix in Eq. (2b)], the reduced-order system [Eq. (2)] may have the outputs directly affected by the inputs [i. e., the term $Du(t)$]. In general, this coefficient is small and may result in only a slight change in the low-frequency behavior of the mode.[1] Without loss of generality, assume that the reduced-order system (2) is controllable and observable. The observer-based controller based on Eq. (2) is assumed to be of the form

$$u(t) = -G\hat{x}_1(t) \tag{3a}$$

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$$\dot{\hat{x}}_1(t) = A\hat{x}_1(t) + Bu(t) + K[\bar{y}(t) - \hat{y}(t)]$$

$$\hat{y}(t) = C\hat{x}_1(t) + Du(t) \tag{3b}$$

where $\hat{x}_1(t)$ and $y(t)$ are, respectively, the estimate of $\bar{x}_1(t)$ and $\bar{y}(t)$, G is the $m \times n_1$ regulator gain matrix, and K is the $n_1 \times r$ observer gain matrix. Combining Eqs. (2) and (3), we can obtain the composite closed-loop system for the reduced-order model [Eq. (2)] as follows:

$$\dot{w}(t) = \Lambda w(t) \tag{4}$$

where

$$\Lambda = \begin{bmatrix} A - BG & -BG \\ 0 & A - KC \end{bmatrix}$$

$w(t) = [\bar{x}_1^T(t)\ e_1^T(t)]^T \in R^{2n1}$ and $e_1(t) = \hat{x}_1(t) - \bar{x}_1(t)$ is the estimator error. Clearly, the gain matrices G and K can be chosen independently. If regulator poles (eigenvalues of $A - BG$) and estimator poles (eigenvalues of $A - KC$) are chosen to be stable, then the matrix Λ is Hurwitz.

It is noted that the gain matrices K and G in Eq. (3) are designed based on the reduced-order system. However, in the actual system, the term $\bar{y}(t)$ in the state estimator [Eq. (3b)] should be replaced by the actual system outputs $y(t)$ [Eq. (1c)]. Now, applying the actually observer-based controller for the full-order model [Eq. (1)], the closed-loop governing equations of Eqs. (1) have the following forms:

$$\dot{z}_1(t) = H_{11}z_1(t) + H_{12}x_2(t) \tag{5a}$$

$$\dot{x}_2(t) = H_{21}z_1(t) + H_{22}x_2(t) \tag{5b}$$

$$y(t) = \tilde{C}_1 z_1(t) + C_2 x_2(t) \tag{5c}$$

where $z_1(t) = [x_1^T(t)\ e_2^T(t)]^T \in R^{2n1}$, the actual estimator error $e_2(t) = \hat{x}_1(t) - x_1(t)$ and the constant matrices

$$H_{11} = \begin{bmatrix} A_{11} - B_1G & -B_1G \\ -\Gamma & A_{11} - KC_1 - \Gamma \end{bmatrix}, \qquad H_{12} = \begin{bmatrix} A_{12} \\ KC_2 - A_{12} \end{bmatrix}$$

$$H_{21} = [A_{21} - B_2G \quad -B_2G], \qquad H_{22} = A_{22}$$

$$\tilde{C}_1 = [C_1 \quad 0], \qquad \Gamma = (KC_2 - A_{12})A_{22}^{-1}(B_2G - A_{21})$$

It is noted that by using the preceding representations and by some calculations, an important relationship may be obtained:

$$\Lambda = H_{11} - H_{12}H_{22}^{-1}H_{21} \tag{6}$$

Although we can select suitable gains G and K such that the reduced-order system is asymptotically stable, this does not guarantee the overall loop stability for the actual system.

We now summarize the preceding descriptions and formulate the specific problem as follows: If A_{22} is nonsingular and both A_{22} and $\Lambda = H_{11} - H_{12}H_{22}^{-1}H_{21}$ are Hurwitz, under what condition can the stability of the actual closed-loop system (5) be deduced from the stable, reduced-order system (4)?

Main Result

Before the main result is depicted, we first give the following lemmas, which will be used in the sequel.

Lemma 1[4]: Let a matrix $E(s) \in S^{m\times n}$ with $S^{m\times n}$ denoting the set of $m \times n$ matrices whose elements are proper stable rational functions, then

$$\sup_{s\in C_+} \|E(s)\|_2 = \sup_{\omega\geq 0}\|E(j\omega)\|_2 \tag{7}$$

where C_+ is the closed, right-half complex plane and $\|E(j\omega)\|_2 = \{\lambda_{max}[E^*(j\omega)E(j\omega)]\}^{1/2} = \bar{\sigma}[E(j\omega)]$ is the largest singular value of $E(j\omega)$ [the notation $\lambda_{max}(F)$ represents the maximum eigenvalue of the matrix F, * denotes the complex conjugate transpose].

Lemma 2: If $E(s) \in S^{n\times n}$ and $\bar{\sigma}[E(j\omega)] < 1$, for all $\omega \geq 0$, then $[I - E(s)]^{-1} \in S^{n\times n}$.

Proof: The spectrum of the matrix $I - E(s)$ consists of $1 - \lambda_i[E(s)]$, $i = 1, 2..., n$, for all $s \in C_+$, in which $\lambda_i[E(s)]$ is the ith eigenvalue of the matrix $E(s)$. By Lemma 1, $\bar{\sigma}[E(j\omega)] < 1$, for all $\omega \geq 0$ implies $\bar{\sigma}[E(s)] < 1$, for all $s \in C_+$. Also, from the fact that for the spectral norm of a matrix P, the following inequality holds[5]:

$$\max_i |\lambda_i(P)| \leq \|P\|_2 = \bar{\sigma}(P)$$

Hence,

$$|\lambda_i[E(s)]| \leq \bar{\sigma}[E(s)] < 1, \qquad \text{for all } i = 1,2,..., n \text{ and } s \in C_+$$

Accordingly, $1 - \lambda_i[E(s)] \neq 0$ for all i and $s \in C_+$, and hence

$$\prod_{i=1}^{n}(1 - \lambda_i[E(s)]) \neq 0$$

if $s \in C_+$. From the preceding analysis, it is clear that $\det[I - E(s)]$ has no zero on the closed, right-half plane. Thus, we have $[I - E(s)]^{-1} \in S^{n\times n}$.

A stable matrix $E(s)$, which satisfies the inequality $\|E(j\omega)\|_2 < 1$ for all $\omega \geq 0$, is called a contractive operator or the class Schur of operator,[6] and the inequality in Lemma 2 is equivalent to [6]

$$D = I - E^*(j\omega)E(j\omega) > 0, \qquad \text{for all } \omega \geq 0 \tag{8}$$

In this equation, the notation matrix $D > 0$ means that D is positive definite.

The following main result is immediately obtained from Lemma 2.

Theorem 1: For the actual closed-loop system [Eq. (5)], and reduced-order, closed-loop system [Eq. (4)], where the matrices $\Lambda = H_{11} - H_{12}H_{22}^{-1}H_{21}$ and H_{22} are Hurwitz, the actual system under control is also asymptotically stable, if

$$\mu_L = \bar{\sigma}[j\omega(j\omega I - \Lambda)^{-1}H_{12}H_{22}^{-1}(j\omega I - H_{22})^{-1}H_{21}] < 1$$

$$\text{for all } \omega \geq 0 \tag{9}$$

Proof: The proof of this theorem is provided in the Appendix.

From Eq. (8), to test the stability condition given in Theorem 1 is equivalent to checking

$$I - \Phi^*(j\omega)\Phi(j\omega) > 0, \qquad \text{for all } \omega \geq 0 \tag{10}$$

where

$$\Phi(j\omega) = j\omega(j\omega I - \Lambda)^{-1}H_{12}H_{22}^{-1}(j\omega I - H_{22})^{-1}H_{21}$$

Therefore, the first step in the controller design is the choice of a regulator gain matrix G in Λ. The gains G are chosen by linear-quadratic or pole-placement criteria. The estimator gains K are then determined such that the eigenvalues of $A - KC$ are placed sufficiently to the left of the spectrum of $A - BG$ for adequate system response and to satisfy the stability condition [Eqs. (9) or (10)].

It is possible to obtain the following inequality, which implies Eq. (9):

$$\bar{\sigma}(H_{12})\bar{\sigma}(H_{22}^{-1})\bar{\sigma}(H_{21}) < \underline{\sigma}(j\omega I - \Lambda)\underline{\sigma}(j\omega I - H_{22})/\omega$$

$$\text{for all } \omega \geq 0 \tag{11}$$

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where $\underline{\sigma}(R) = [\bar{\sigma}(R^{-1})]^{-1}$ denotes the smallest singular value of the nonsingular matrix R. In most large flexible structures, the coupling matrices A_{12} and A_{21} in Eq. (1) are zero. Using this and $H_{22} = A_{22}$, we have

$$\bar{\sigma}(H_{12}) = \bar{\sigma}(KC_2), \quad \bar{\sigma}(H_{21}) = \sqrt{2}\bar{\sigma}(B_2G), \quad \bar{\sigma}(H_{22}^{-1}) = \bar{\sigma}(A_{22}^{-1})$$

then Eq. (11) can be reformulated as

$$\beta < \inf_{\omega \geq 0} [\underline{\sigma}(j\omega I - \Lambda)\underline{\sigma}(j\omega I - A_{22})/\omega] \tag{12}$$

where $\beta = \sqrt{2}\bar{\sigma}(KC_2)\bar{\sigma}(B_2G)\bar{\sigma}(A_{22}^{-1})$

It should be pointed out that the stability condition presented in Theorem1 is a sufficient condition. That is, even if Eq. (9) does not hold, we cannot say that no stabilizing controller exists. However, if the chosen gain matrices G and K can satisfy this condition, then the overall system is guaranteed to be stable.

Robust Stability

In general, there will be many G, K that satisfy Eq. (9). It is proposed that the additional freedom should be used to provide the robustness of stability to the structured plant uncertainties of Eq. (1). For the typical flutter suppression problem or structural control problem, the additional plant uncertainties arise from mass variations caused by fuel, wing stores, or possible structural variations.

Based on Eq. (5), suppose that the perturbed full-order system being controlled is given by

$$\dot{h}(t) = (\Psi + \Delta\Psi)h(t) \tag{13}$$

where

$$\Psi = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}, \quad \Delta\Psi = \Delta\Psi(\Delta A_{11}, \Delta A_{12}, \Delta A_{21}, \Delta B_1, \Delta C_1,)$$

and $h(t) = [x_1^T(t)\ \hat{x}_1^T(t)\ e_2^T(t)]^T \in R^{2n1+n2}$; $\Delta\Psi$ is a perturbation matrix. Assume only the norm of $\Delta\Psi$ is known and it is $\bar{\sigma}(\Delta\Psi)$, and the set of permissible G and K has been chosen such that the stability condition (9) holds (so that the matrix Ψ is Hurwitz). The perturbed system is asymptotically stable if some of the gain matrices G and K in this set can make

$$\bar{\sigma}(\Delta\Psi) < \underline{\sigma}(j\omega I - \Psi), \qquad \text{for all } \omega \geq 0 \tag{14}$$

hold.[7] That is, if the gains G and K can be chosen to satisfy Eqs. (9) and (14), we will achieve robust stability for the perturbed full-order system.

Example

To demonstrate the accuracy and reliability of our main result, we consider a simply supported beam example.[8] The first four modes of this beam are viewed as the controlled modes, whereas the fifth and sixth modes act as the residual modes. The beam damping factor is 0.01. All nominal position states are initially set to 1.0, whereas all nominal velocity states and all residual states are initially set to zero. We do not consider the situation where the sensor and actuator are collocated; thus the actuator is assumed to be located at the 1/6 position and the displacement sensor is located at the 5/6 position. An observer-based controller was adopted to control this beam. To achieve a satisfactory performance, the regulator gains were selected as

$G =$

$[-0.016\ 0.273\ 2.348\ 6.856\ 0.036\ 0.132\ 0.264\ 0.371] \times 10^3$

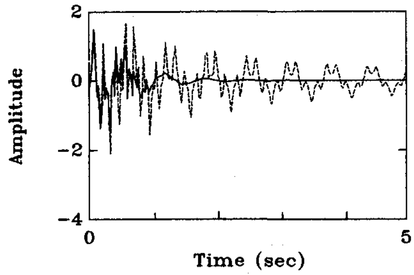

Fig. 1 Controlled (solid line) and uncontrolled beam deflections for four-controlled-mode model.

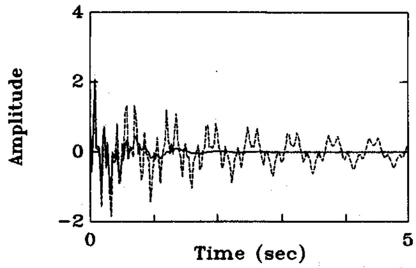

Fig. 2 Controlled (solid line) and uncontrolled beam deflections for three-controlled-mode model.

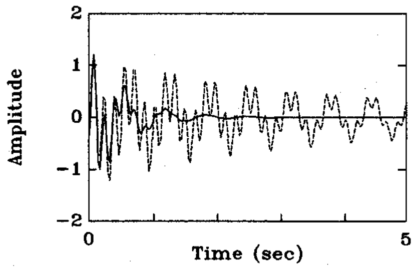

Fig. 3 Controlled (solid line) and uncontrolled beam deflections for two-controlled-mode model.

and the estimator gains,

$K =$

$[0.187\ -0.182\ 0.169\ -0.151\ -0.258\ -0.666\ 3.387\ -7.228]^T$

$\times 10^2$

The maximum μ_L (robustness measure) in Eq. (9) over all frequencies was found as 0.20, i. e., the stability condition of Eq. (9) is satisfied, and there still remains a sufficiently large stability margin for the controlled system to tolerate more residual modes. The output deflections for the controlled and uncontrolled beams are shown in Fig. 1. We also expand the preceding result to the case when only the first three modes were controlled. We choose

$$G = [-0.015 \quad 0.309 \quad 2.845 \quad 0.036 \quad 0.132 \quad 0.264] \times 10^3$$

$$K = [0.187 \quad -0.181 \quad 0.166 \quad -0.248 \quad -0.825 \quad 4.417]^T \times 10^2$$

and find the maximum $\mu_L = 0.38$. As only the first two modes were controlled, we choose

$$G = [-0.128 \quad 5.0 \quad 0.355 \quad 1.316] \times 10^2$$

$$K = [0.188 \quad -0.178 \quad -0.210 \quad -1.549]^T \times 10^2$$

and find the maximum $\mu_L = 0.49$. In the latter two cases, the selected gains also provide satisfactory performance, and the controlled and uncontrolled beam deflections are shown in Figs. 2 and 3, respectively.

Conclusions

In this Note, we consider the stabilization of large, flexible structures with mode residualization. A new frequency domain stability condition for a high-order structure controlled by a reduced-order observer-based controller is derived. As linear parametric perturbations are involved in the controlled structure, an additional stability condition that guarantees robust stability for the perturbed structure is also presented.

Appendix: Proof of Theorem 1

Taking Laplace transform of the state-space representations of Eq. (5) yields

$$sZ_1(s) = H_{11}Z_1(s) + z_1(0) + H_{12}X_2(s) \quad \text{(A1a)}$$

$$X_2(s) = (sI - H_{22})^{-1}H_{21}Z_1(s) + (sI - H_{22})^{-1}x_2(0) \quad \text{(A1b)}$$

$$Y(s) = \tilde{C}_1Z_1(s) + C_2X_2(s) \quad \text{(A1c)}$$

where $z_1(0)$ and $x_2(0)$ are, respectively, the initial states of $z_1(t)$ and $x_2(t)$. Substituting Eq. (A1b) into Eq. (A1a), and after some manipulations, we get

$$Z_1(s) = K(s)^{-1}z_1(0) + K(s)^{-1}H_{12}(sI - H_{22})^{-1}x_2(0) \quad \text{(A2)}$$

where $K(s) = sI - H_{11} - H_{12}(sI - H_{22})^{-1}H_{21}$. $K(s)$ can be further expressed as

$$\begin{aligned} K(s) &= sI - H_{11} + H_{12}H_{22}^{-1}H_{21} - H_{12}H_{22}^{-1}H_{21} \\ &\quad - H_{12}(sI - H_{22})^{-1}H_{21} \\ &= sI - \Lambda - H_{12}H_{22}^{-1}[I - (I - sH_{22}^{-1})^{-1}]H_{21} \\ &\qquad (\text{where } \Lambda = H_{11} - H_{12}H_{22}^{-1}H_{21}) \\ &= (sI - \Lambda)[I - s(sI - \Lambda)^{-1}H_{12}H_{22}^{-1}(sI - H_{22})^{-1}H_{21}] \end{aligned} \quad \text{(A3)}$$

Substituting Eq. (A3) into Eq. (A2), we obtain

$$\begin{aligned} Z_1(s) &= [I - s\,(sI - \Lambda)^{-1}H_{12}H_{22}^{-1}(sI - H_{22})^{-1}H_{21}]^{-1} \\ &\quad \times (sI - \Lambda)^{-1}z_1(0) + [I - s(sI - \Lambda)^{-1}H_{12}H_{22}^{-1} \\ &\quad \times (sI - H_{22})^{-1}H_{21}]^{-1}(sI - \Lambda)^{-1}H_{12}(sI - H_{22})^{-1}x_2(0) \end{aligned} \quad \text{(A4)}$$

Since

$$(sI - \Lambda)^{-1} \in S^{2n1 \times 2n1}, \qquad (sI - H_{22})^{-1} \in S^{n2 \times n2}$$

$$s(sI - \Lambda)^{-1}H_{12}H_{22}^{-1}(sI - H_{22})^{-1}H_{21} \in S^{2n1 \times 2n1}$$

by Lemmas 1 and 2, we therefore know that if

$$\bar{\sigma}[j\omega(j\omega I - \Lambda)^{-1}H_{12}H_{22}^{-1}(j\omega I - H_{22})^{-1}H_{21}] < 1, \quad \text{for all } \omega \geq 0$$

holds, then $z_1(t)$, the inverse Laplace transform of $Z_1(s)$ in Eq. (A4), is asymptotically stable. From Eq. (A1b), since $(sI - H_{22})^{-1} \in S^{n2 \times n2}$ and $z_1(t)$ and $x_2(t)$ have been stabilized. Since both $z_1(t)$ and $x_2(t)$ are stabilized, obviously, $y(t)$ is also asymptotically stable.

References

[1]Takahashi, M., and Slater, G. L., "Design of a Flutter Mode Controller Using Positive Real Feedback," *Journal of Guidance, Control, and Dynamics,* Vol. 9, 1986, pp. 339-345.

[2]Bossi, J. A., and Price, G. A., "A Flexible Structure Controller Design Method Using Mode Residualization and Output Feedback," *Journal of Guidance, Control, and Dynamics,* Vol. 7, 1984, pp. 125-127.

[3]Lynch, P. J., and Banda, S. S., "Active Control for Vibration Damping," *Large Space Structures: Dynamics and Control,* edited by S. N. Atluri, Springer, New York, 1988.

[4]Vidyasagar, M., *Control System Synthesis: A Factorization Approach,* MIT Press, Cambridge, MA, 1985.

[5]Desoer, C. A., and Vidyasagar, M., *Feedback Systems: Input-Output Properties,* Academic, New York, 1975.

[6]Delsarte, P., Genin, Y., and Kamp, Y., "Schur Parametrization of Positive Definite Block-Toeplitz Systems," *SIAM Journal on Applied Mathematics,* Vol. 36, 1979, pp. 34-46.

[7]Qiu, L., and Davison, E. J., "New Perturbation Bounds for the Robust Stability of Linear State Space Modes," *Proceedings of the 25th IEEE Conference on Decision and Control,* Athens, Dec. 1986, pp. 751-755.

[8]Bar-Kana, I., Kaufman, H., and Balas, M., "Model Reference Adaptive Control of Large Structural Systems," *Journal of Guidance, Control, and Dynamics,* Vol. 6, 1983, pp. 112-118.

Effect of Thrust/Speed Dependence on Long-Period Dynamics in Supersonic Flight

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Nomenclature

$A(s)$	= coefficient matrix of the homogenous system
B	= scaling matrix for control inputs
C_D	= drag coefficient
C_{D_u}	= $\partial C_D/\partial(u/V_0)$
C_{D_α}	= $\partial C_D/\partial\alpha$
C_L	= lift coefficient
C_{L_u}	= $\partial C_L/\partial(u/V_0)$
C_{L_α}	= $\partial C_L/\partial\alpha$
C_m	= pitching moment coefficient
C_{m_q}	= pitch damping, $= 2\partial C_m/\partial(q\bar{c}/V_0)$
C_{m_α}	= $\partial C_m/\partial\alpha$
$C_{m_{\dot\alpha}}$	= $2\partial C_m/\partial(\dot\alpha\bar{c}/V)$
$\bar{c}$	= mean aerodynamic chord
g	= acceleration due to gravity
h	= altitude
i_y	= radius of gyration
k_ρ	= $-(g/V_0^2)/\rho_h$
M	= Mach number
n_u	= thrust/speed dependence, $= (V_0/T_0)\partial T/\partial u$
$\bar{q}$	= dynamic pressure, $= (\rho/2)V^2$
S	= reference area
s	= Laplace operator
T	= thrust
u	= speed perturbation
V	= airspeed
x	= variable vector
α	= angle of attack
δ_T	= thrust setting
μ	= relative mass parameter, $= 2m/(\rho S\bar{c})$
ρ	= air density
ρ_h	= density gradient, $= (1/\rho_0)\, d\rho/dh$
σ	= real part of complex variable

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